Multi Channel Wave Transfer in Sparsely Connected Systems

Rudolf Sprik

Van der Waals-Zeeman Instituut, Universiteit van Amsterdam, Amsterdam, The Netherlands (e-mail: sprik@science.uva.nl)

Abstract. Wave transport in finite systems connected through multiple channels is strongly influenced by inter channel correlations. Such correlations are induced by scattering and restrictions in the complex medium in which the waves propagate. Examples can be found in electron conduction in mesoscopic systems with multiple leads and in electromagnetic wave transfer in multi-antenna wireless communication. The influence of the correlations on the overall wave transport can be uncovered by analyzing the eigenvalue distributions of the transfer matrix between the ingoing and outgoing channels. A model system is studied experimentally in the microwave region where the correlation between sets of transmitting and receiving antennas are induced by connecting complex cavities through a limited number of leads. The eigenvalue distribution of this correlated multi channel system is described using random matrix theory.
Keywords: Random matrix modeling, Eigenvalue distribution, Classical wave transport, Coupled chaotic systems.

1 Introduction

Wave transport in a strongly scattering environment occurs in many systems. Examples are found in classical wave transport of light, electromagnetic waves, sound and elastic waves[1]. Also in quantum mechanical problems such as electronic transport in small devices multiple scattering is frequently encountered. In the past decades the effects of multiple scattering on wave transport has been studied intensely both experimentally and theoretically. E.g. scattering theory developed for quantum field problems has been successfully transferred to solve problems in classical and quantum mechanical wave scattering and lead to understanding of wave scattering related phenomena such as weak and strong localization.

In recent developments of wireless communication in strongly scattering indoor (e.g. office spaces) and outdoor (e.g. cities) environments the scattering is exploited to improve the transfer[2]. The advantage of multiple antennas on transmit and receive side relies on scattering induced by the environment. The theoretical framework to analyze the wave transfer in strongly scattering and noisy systems has been developed in the past decade as an important topic in information theoretical studies[3] starting with the work of Shannon in 1948[4].

The details of the underlying mechanisms of scattering on transport in wireless communication or of the Hamiltonian of a (quantum) mechanical system are not required to model a system. A powerful approach is provided by random matrix theory (RMT[5]). The random matrix approach was started by Wigner[6] to describe distributions of quantum mechanical eigenvalues of complex systems and found early applications in the field of mathematical statistics by Wishart[7]. In the information theory of wireless communication RMT has been applied with success to a variety of multi-channel situations without and with correlations between channels[8]. The elements in the transfer matrix H describing the phase and amplitude at the receiving antennas due to a signal at the transmitting antennas can be modelled by statistically defined distributions[9]. The eigenvalues of $HH^{\dagger}$, with $H^{\dagger}$ the Hermitian adjoint of H, are an essential ingredient in estimating the transfer capacity of a multi channel connection. For example for random matrices with independent and identically distributed elements Marčenko and Pastur[10] derived general analytical expression for the eigenvalue distributions applicable to transmissions between rectangular antenna arrays. Once the properties of the eigenvalue distributions are known, an estimate of the transfer capacity and the fluctuations can be made. Finding the eigenvalues λ_i^2 of $HH^{\dagger}$ is equivalent to finding the singular values λ_i of H, which is sometimes more convenient. The generalized Shannon expression for the transfer capacity is[9]: $C_{shannon} = -\log\det[I + \eta HH^{\dagger}]$, with η the signal to noise ratio in the system, I the unit matrix and depends intricately on the eigenvalue distribution.

Correlation between channels decreases the transfer capacity considerably. The effect of correlation induced by constrictions in space between transmitter and receiver in the form of keyholes or waveguides has been the topic of mostly theoretical [11–13] and some experimental studies[14,15]. Such constrictions are found typically in transfer between floors in buildings and urban environments.

In this article a controllable model system with classical waves in two coupled complex and dissipative closed cavities will be used to study the influence of correlations between channels on wave transport. The experiments described in Sections 2 and 2.2 enable a systematic variation of the induced correlation and the influence on resonances in the wave transport. The results are analyzed with a random matrix approach in Section 3.

2 Wave Transfer in Sparsely Coupled Systems

2.1 Experiments

The microwave experiment in the 0.5-2 GHz band (i.e. 30-15 cm wavelength) uses two almost chaotic cavities (see Figure 1). Within each of the cavities the reflections from the walls and from the enclosed antennas causes an almost chaotic transport of the waves from multiple antennas to a limited number antennas connected to outgoing channels. These outgoing channels are then launched into the second cavity and propagate to the receiving antennas.

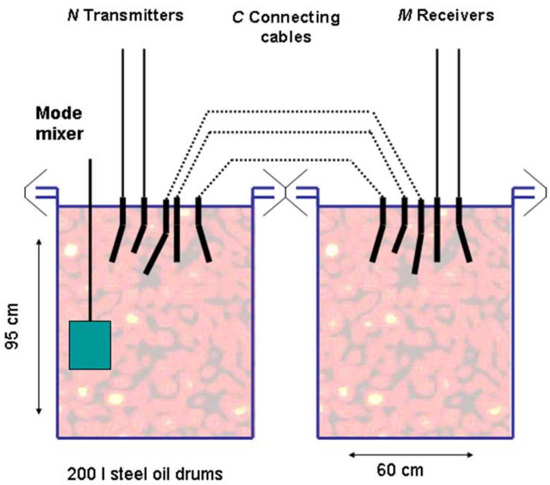

Fig. 1. Layout of a multi-channel microwave experiment with microwaves in the 0.5-2 GHz band. N transmitting antennas in a closed cavity are sending the signal through C connecting cables to a second closed cavity with M receiving antennas.

The receiver and transmitter antennas are 2.4 GHz omni-directional antennas on a swivel mount used in e.g. computer WiFi connections and are fitted into the lids of two 200 liter steel oil drums. Some of the antennas at the transmitter side are connected by coax cables directly to antennas at the receiver side. The overall quality factor of the drum cavity is $Q \approx 300$. The transfer as function of frequency between the M transmitting and N receiving antennas is determined by measuring the S-parameters S_{21}, S_{11}, S_{12}, and S_{22} with an Agilent 4396B network analyzer in the 0.5GHz - 1.8GHz range. The S-parameters between two ports are defined by the ratio of the voltage of the reflected and the transmitted wave with respect to the voltage of the incoming wave in a 50Ω terminated system. E.g. S_{21} characterizes the transmitted and S_{11} the reflected wave for a signal injected into port 1 and received at port 2. In a dissipative system $1 - |S_{12}|^2 + |S_{11}|^2$ gives the dissipation and in a reciprocal system $S_{21} = S_{12}$. A homemade coaxial multiplexing switch selects all possible transmit-receive combinations to obtain the multi-channel scattering matrix S_{ij} with $i = 1, ..., N$ and $j = 1, , M$ and gives the full $M \times N$ complex transfer matrix H of the system as function of frequency. The connecting paths for the different switch selections are identical and all unused antennas are terminated by a 50Ω impedance.

Calibration of the S-parameter measurement is performed by fitting the 12 parameter correction model[16]. In the reference measurements the antenna connections are terminated with a short, an open, a 50Ω load, and a direct through connection (SOLT calibration model). The calibration procedure eliminates the influence of the leads and switch contacts and gives the true multiport S-parameters between the antennas. Averaging over configurations is achieved by the rotation of a copper plate inside the oil drum ('mode mixer' see figure 1). The set up is under computer control to enable signal averaging over configurations and perform multiplexing of a bi-directional measurement of an antenna array with a maximum of 8 transmitters and 8 receivers.

2.2 Experimental results

Figure 2 shows the typical observed variations in the matrix elements for the uncorrected S-parameters in a 3 transmitter × 3 receiver configuration with 2 connecting cables between the drums.

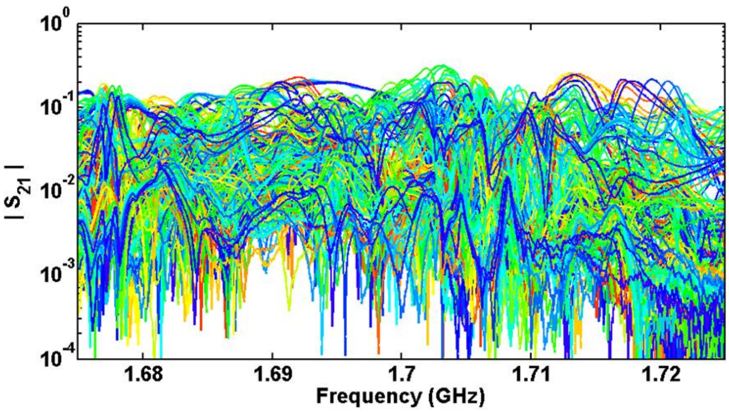

Fig. 2. The different curves combine all the uncorrected S_{21} measurements with a $N = 3$, $C = 2$, and $M = 3$ configuration for 21 different positions of the 'mode mixer' in 50MHz band around 1.7GHz.

After correction the data is used to extract the statistics of the elements in $H(\omega)$ and are shown in Figure 3. The distribution shows a uniform coverage in phase, i.e. $< H(\omega) >= 0$. The distribution of the amplitude is not

Gaussian but is characteristic for the fading behavior in frequency for a set of well separated narrow resonances with resonance frequencies distributed over the frequency interval and with varying strengths and widths as shown in Fig. 2).

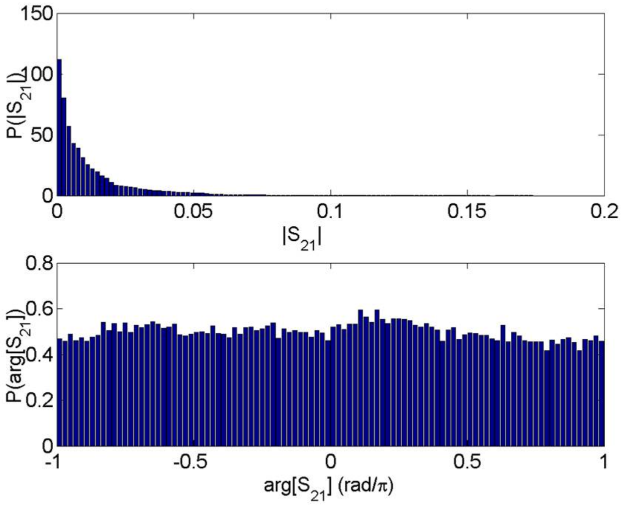

Fig. 3. Distribution of amplitude and phase angle of the corrected transmission parameter S_{21} between the different transmit and receive antennas with $C = 1$.

The distribution of the singular values of $H(\omega)$ are given in Figure 4 for different number of connecting cables ($C = 1, 2$, and 3). The distribution for $C = 1$ show a strong peak at 0 and a suppressed probability close to zero. The distributions with $C > 1$ look very similar. The distributions show a distinct structure that can be associated with the direct line of sight contact between antennas.

3 Modelling the Results with Random Matrices

3.1 Random matrix model for coupled systems

The transfer matrix H between the antennas in drum 1 to the receiving antennas in drum 2 will be modelled by a product of random matrices:

$$H = TCX. \tag{1}$$

Here matrix T is a $N \times C$ dimensioned and matrix X a $C \times M$ dimensioned random matrix with identical (independent) distributed elements. The deterministic matrix T represents the transfer between the M antennas on the

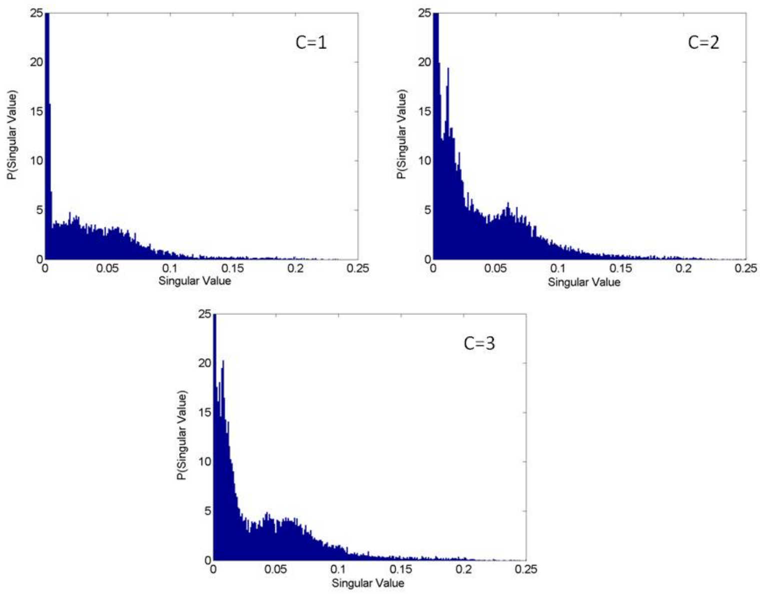

Fig. 4. Singular value distributions of $H(\omega)$ with $N = M = 3$ receiving and transmitting antennas and $C = 1, 2, 3$ interconnections averaged over all 'mode mixer' positions and frequencies.

transmitter to the C antennas connected to the cables between the transmitter and receiver side. Likewise X represents the connection from the C cables to the M receiving antennas. The connection matrix C is a diagonal $C \times C$ matrix representing the transfer through the C connecting cables between the transmitter and the receiver side.

3.2 Independent transmit and receive channels

The simplest approach is to assume that the random elements in T and X are independent identical complex distributions with random phase and equal amplitude (Gaussian distribution). Simulation results for the matrix elements of H and the eigenvalue distribution of $HH^\dagger$ are shown in Figure 5 for 5000 realizations and increasing number of links. When the rank$(C) >$ max[rank(M, N)] the distributions quickly converge to the same behavior for the elements of H and for the eigenvalue distribution.

Müller[17] derived the asymptotic eigenvalue distribution for products of random matrices in the limit of very large matrix sizes as a generalization of the Marčenko-Pastur result[10]. The current examples are not in that limiting regime yet. However, the convergence to the asymptotic limit is in general approached fast for very small systems already[8]. Including the

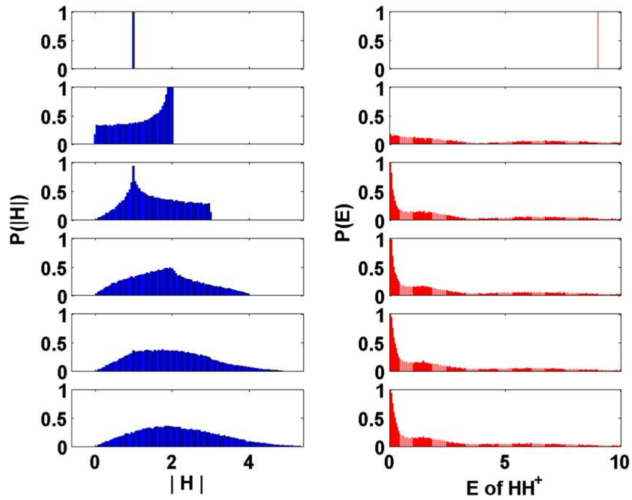

Fig. 5. Random matrix simulation of the eigenvalue distribution of $HH^\dagger$ and matrix elements for the random transfer matrix H with $M = 3$ transmitters and $N = 3$ receivers connected through C restricting links. The elements of T and X are independently identically distributed complex numbers with random phase and unit amplitude. Coupling matrix C is the unit matrix for $C = 1$ to 6 from top to bottom.

fading behavior as function of frequency indicated by Fig. 2 and Fig. 3 does not influence the asymptotic behavior for uncorrelated matrix elements.

3.3 Correlations on the transmit and receive side

In the experiment there are different causes for correlations between channels. The connecting cables restrict the transfer to a finite number of channels and cause correlation as shown in the previous paragraph. In addition, the antennas on the receiver and transmitter side are placed close enough to be correlated due to the finite extent of coherent regions in the wave pattern inside the drum. Analysis of the signals from the transmitters on one side to the connecting antennas in the same drum show indeed correlation inside each of the drums. Using the statistics of the transfer within the drums in combination with the effects of the extra links will not be pursued in detail here. Given the limited number of configurations included in the estimates of the distributions, it is difficult to sample enough configurations to be able to use the random matrix approach.

4 Conclusions

The random matrix theory is not describing the observed distribution in detail. Improvement can be obtained by averaging over more antenna configurations and choosing a geometry where near field and direct coupling is suppressed.

References

1.P. Sheng, 'Introduction to Wave Scattering, Localization, and Mesoscopic Phenomena', Academic Press, 1995.
2.S.H. Simon, A.L. Moustakas, M. Stoytchev, and H. Safar, Communication in a Disordered World, Physics Today, september:38 (2001).
3.T. M. Cover and J. A. Thomas, 'Elements of Information Theory', John Wiley & Sons, Inc, 1991.
4.C.E. Shannon, A mathematical theory of communication, Bell Syst. Tech. J., 27:379 and 623, 1948.
5.C.W.J. Beenakker, Random-matrix theory of quantum transport, Rev. Mod. Phys. 69:731, 1997.
6.E. Wigner, On the distribution of the roots of certain symmetric matrices, Ann. Math. 67: 325-327 (1958).
7.J. Wishart, The generalized product moment distribution in samples from a normal multivariate population, Biometrika 20: 35-52 (1928).
8.A. Tulino and S. Verd, Random Matrix Theory and Wireless Communications, Found. Trends Commun. Inf. Theory , 1:1, 2004.
9.V. A. Marčenko and L. A. Pastur, Distributions of eigenvalus of some sets of random matrices, Math. USSR-Sbornik, 1:457, 1967.
10.E. Telatar, Capacity of multi-antenna gaussian channels , AT & T-Bell Technical Memorandum (1995, see: http://mars.bell-labs.com); Eur. Trans. Telecom., 10:585, 1999.
11.R. Sprik, Wireless transfer between antenna arrays at the Talbot distance, Phys. Rev. E, 72:037602, 2005.
12.D. Chizhik, G. Foschini, M. Gans, and R. Valenzuela, Keyholes, Correlations, and Capacities of Multielement Transmit and Receive Antennas, IEEE Trans. Commun., 1:361, 2002.
13.S. Loyka and A. Kouki, On MIMO Channel Capacity, Correlations and Keyholes: Analysis of Degenerate Channels, IEEE Trans. on Comm., 50:1886, 2002.
14.D. Chizhik, J. Ling, P. W. Wolniansky, R. A. Valenzuela,, N. Costa, K. Huber, Multiple-Input-Multiple-Output Measurements and Modeling in Manhattan, IEEE Journal on Sel. Areas in Comm., 21:321 , 2003.
15.P. Almers, F. Tufvesson, and A. F. Molisch, Measurement of Keyhole Effect in a Wireless Multiple-Input Multiple-Output (MIMO) Channel, IEEE Comm. Lett., 7:373, 2003.
16.Agilent application note AN 1287-3, Applying Error Correction to Network Analyzer Measurements, 2002 (see: http://www.agilent.com).
17.R.R. Müller, A random matrix model of communication via antenna arrays, IEEE Trans. Inform. Theory, 9:2495, 2002.