LARGE EDDY SIMULATION OF COMPRESSIBLE MHD TURBULENCE IN SPACE PLASMA

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Abstract. Large eddy simulation (LES) method for study of compressible magnetohydrodynamic (MHD) turbulence is developed. The filtered equations of magnetohydrodynamics of compressible fluid are obtained with the use of mass-weighted filtering procedure (Favre filtering). Favre-filtered equations for large-scale component of turbulence include subgrid-scale terms describing subgrid phenomena. Different models for closure of subgrid terms are suggested. In this work numerical simulation of filtered MHD equations and an analysis of the received characteristics of turbulent flow are carried out. The obtained results of numerical computations for different LES models are compared with the results of direct numerical simulation (DNS).
Keywords: Turbulence, Simulations, Subgrid-scale motions, MHD.

Introduction

Numerical simulation of turbulent magnetohydrodynamic (MHD) flows is an effective tool for the study of the flows of the charged fluid of astrophysical, helio- and geophysical plasma (for example, solar corona expansion, solar wind, flows in the solar convection zone, turbulence in interstellar matter), which is inaccessible for direct experimental study.

To overcome the Reynolds number limitation of direct numerical simulations (DNS), imposed by the finite capabilities of available computational resources, the large eddy simulation (LES) technique can be applied. LES approach describes approximate turbulence dynamics, where the large-scale part of turbulent flow is computed directly, while the small-scale one is modeled.

LES method has yet to apply for compressible magnetohydrodynamic turbulent flows, all the previous works in this direction were limited by consideration of incompressible fluid only (see [2]). In [2] LES is used for study of incompressible MHD turbulent flow. This was done by extending known hydrodynamic closures for magnetohydrodynamic case and proposing new subgrid-scale models. In the present work the large eddy simulation is generalized for the study of compressible magnetohydrodynamic turbulence for the first time. It is assumed that the relation between density and pressure is polytropic. This assumption about polytropic process is used to study and simulate compressible turbulence of neutral and magnetized fluids, turbulence of the solar wind, interstellar turbulence as well as other problems of astrophysical turbulence.

Favre-filtered equations

To simplify equations describing turbulent MHD flow with variable density it is convenient to use Favre filtering (known as mass-weighted filtering) so as to avoid appearance of additional SGS-terms. Mass-weighted filtering is used for all parameters of charged fluid flow besides the pressure and magnetic fields. Mass-weighted filtering is determined as follows: $\tilde{f} = \overline{\rho f}/\bar{\rho}$. Filtering is designated by two symbols, namely, the overline designates ordinary filtering, while the tilde specifies mass-weighted filtering.

filtered continuity equation

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho}\tilde{u_j}}{\partial x_j} = 0 \tag{1}$$

filtered momentum conservation equation

$$\frac{\partial \bar{\rho}\tilde{u_i}}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{\rho}\tilde{u_i}\tilde{u_j} + \bar{p}\delta_{ij} - \frac{1}{Re}\ \tilde{\sigma_{ij}}\right) + \left(\frac{\bar{B}^2}{2M_a^2}\delta_{ij} - \frac{1}{2M_a^2}\ \bar{B}_j\bar{B}_i\right) = -\frac{\partial \tau_{ji}^u}{\partial x_j} \tag{2}$$

filtered induction equation

$$\frac{\partial \bar{B}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\tilde{u_j}\bar{B}_i - \tilde{u_i}\bar{B}_j\right) - \frac{1}{Re_m}\frac{\partial^2 \bar{B}}{\partial x_j^2} = -\frac{\partial \tau_{ji}^b}{\partial x_j} \tag{3}$$

where ρ - density; p - pressure; u_j - velocity in direction x_j; $\sigma_{ij} = 2\mu S_{ij} - \frac{2}{3}\mu S_{kk}\delta_{ij}$ - viscous stress tensor; $S_{ij} = 1/2\left(\partial u_i/\partial x_j + \partial u_j/\partial x_i\right)$ - strain rate tensor; μ - viscosity; δ_{ij} - the Kronecker delta; η - magnetic diffusion; σ - specific electric conductivity; B - magnetic field; Re - Reynolds number; Re_m - magnetic Reynolds number; M_s - Mach number; M_a - magnetic Mach number.

To close the system of equations it is assumed that the relation between density and pressure is polytropic and has the following form: $p = \rho^\gamma$, where γ is a polytropic index and it is supposed that $\gamma = 5/3$. In the right-hand members of equations (2) - (3) the terms designate influence of subgrid terms on the filtered part:

$$\tau_{ij}^u = \bar{\rho}\left(\widetilde{u_i u_j} - \tilde{u_i}\tilde{u_j}\right) - \frac{1}{M_a^2}\left(\overline{B_i B_j} - \bar{B}_i\bar{B}_j\right) \tag{4}$$

$$\tau_{ij}^b = \left(\overline{u_i B_j} - \tilde{u_i}\bar{B_j}\right) - \left(\overline{B_i u_j} - \bar{B_i}\tilde{u_j}\right) \quad (5)$$

Thus, the filtered system of magnetohydrodynamic equations contains the unknown turbulent tensors: τ_{ij}^u and τ_{ij}^b. To determine their components special turbulent closures (parameterizations) based on large-scale values describing turbulent magnetohydrodynamic flow must be used.

Subgrid modeling

Let us assume that the turbulent tensor τ_{ij}^u is connected to the strain rate tensor and viscosity (eddy viscosity model), while τ_{ij}^b is connected to the dissipation because of resistance (i.e. this dissipation is expressed through the generalized Ohm's law and is equal to ηj, where j is the electric current density) in the following way:

$$\tau_{ij}^u - \frac{1}{3}\tau_{kk}^u\delta_{ij} = -2\nu_t\left(\tilde{S_{ij}} - \frac{1}{3}\tilde{S_{kk}}\delta_{ij}\right) \quad (6)$$

$$\tau_{ij}^b - \frac{1}{3}\tau_{kk}^b\delta_{ij} = -2\eta_t\bar{J_{ij}} \quad (7)$$

where $\bar{J_{ij}} = 1/2\left(\partial\bar{B_i}/\partial x_j - \partial\bar{B_j}/\partial x_i\right)$is a large-scale magnetic rotation tensor; ν_t and η_t are scalar turbulent functions depending on spatial coordinates and time.

Smagorinsky model for MHD-turbulence

The Smagorinsky model is an eddy viscosity model, where subgrid scales are assumed to be isotropic and are in balance with large-scale flow:

$$\nu_t = C_s\bar{\rho}\bar{\triangle}^2|\tilde{S^u}|, \quad (8)$$

$$|\tilde{S^u}| = \left(2\tilde{S_{ij}^u}\tilde{S_{ij}^u}\right)^{1/2}; \qquad \bar{\triangle} = \left(\bar{\triangle}_x\bar{\triangle}_y\bar{\triangle}_z\right)^{1/3}$$

$$\eta_t = D\bar{\triangle}^2|j|. \quad (9)$$

The realizability conditions allows to find subgrid scale model for the turbulent kinetic energy k corresponding to the eddy viscosity model[1].

$$k \geq \frac{1}{2}\sqrt{3}C_s\bar{\rho}\bar{\triangle}^2|\tilde{S}^u|^2 \quad (10)$$

Then it follows the subgrid closure for the isotropic term:

$$\tau_{kk}^u = 2C_I\bar{\rho}\bar{\triangle}^2|\tilde{S^u}|^2 \quad (11)$$

For other subgrid models presented below isotropic terms are determined similarly. Non-universality of constants C_s, C_I and D is determined by the dynamic procudure for all SGS models in our work.

Kolmogorov model for MHD-turbulence

If the filter length is in the inertial subrange of the completely developed turbulence, the kinetic subgrid-energy and the magnetic subgrid-energy dissipation can be assumed only time dependent and constant in space. These parameterizations are based on the Kolmogorov scaling model:

$$\nu_t = C_s \bar{\rho} \bar{\triangle}^{4/3} \tag{12}$$

$$\eta_t = D \bar{\triangle}^{4/3} \tag{13}$$

Model based on cross-helicity for MHD-turbulence

Let us define the cross-helicity in the following as: $H^c = \int_V (u \cdot B) dV$. In MHD-turbulence the typical values of turbulent velocity and magnetic field and, correspondingly, cross-helicity taking into account topology of magnetic field and velocity field as well as space location are found. The cross-helicity is related to the transfer between kinetic and magnetic energies caused by the Lorentz force [2]. Therefore, the cross-helicity allows to estimate energy exchange between large and small scales in the LES method:

$$\nu_t = C_s \bar{\rho} \bar{\triangle}^2 |\tilde{S^u_{ij}} \ \bar{S^b_{ij}}|^{1/2} \tag{14}$$

$$\eta_t = D \bar{\triangle}^2 sgn(\bar{j} \ \tilde{\omega}) |\bar{j} \ \tilde{\omega}|^{1/2} \tag{15}$$

Here, $\bar{S^b_{ij}} = \left(\partial \bar{B}_i / \partial x_j + \partial \bar{B}_j / \partial x_i\right)/2$, $\tilde{\omega} = \nabla \times \tilde{u}$ and function $sgn(.)$ determines the sign of argument. In the model based on cross-helicity only the magnetic diffusion coefficient η_t can change its sign, because in the magnetohydrodynamic turbulence only magnetic energy responds for process of transfer from small to large scales [2].

Numerical results

In this work all simulations are made for decaying compressible MHD turbulence. The results obtained with LES are compared with DNS computations. Uniform mesh with 32^3 grid cells is used for LES and 150^3 for DNS. Test computations and comparison of the results with by using different types of closures are carried out for $Re = Re_m = 75$, $M_s = M_a = 0.5$. Since the numerical code is based on the second order accuracy non-spectral finite-difference schemes for MHD equations written in the conservative form, for separating the turbulent flow into large and small eddy components, Gaussian filter or top-hat filter is applied. In the work [3] Sagaut and Grohens proved the coincidence of optimal discrete forms of Gaussian filter and top-hat filter for central finite-difference scheme of second order of accuracy. It makes the central-difference code independent of the choice of filters. For time

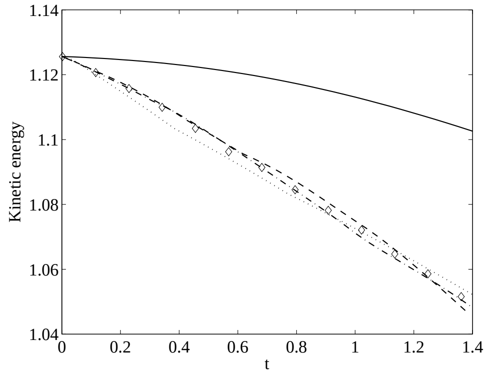

Fig. 1. Time evolution of the kinetic energy. Diamond line - DNS results; solid line - LES results without subgrid closures; dot line - Kolmogorov model; dashed line - Smagorinsky model; dash-dot line - cross-helicity model.

integration the 4th order Runge-Kutta method is used. Periodic boundary conditions for all the three dimensions are applied, the simulation domain is a cube with dimensions of $2\pi \times 2\pi \times 2\pi$. Time evolution of the filtered kinetic energy is shown on Fig. 1, the evolution of filtered magnetic energy is shown on Fig. 2, i.e. Fig. 1 and Fig. 2 show the effect of the presented parameterizations on the resolved kinetic and magnetic energy during turbulence decay. Diamond line represents DNS-results and are compared with different SGS models. Solid line is LES results without any closure, i.e. the tensors τ_{ij}^u and τ_{ij}^b are omitted in filtered equations of magnetohydrodynamics. The dot line represents Kolmogorov model, the dash-dot line represents cross-helicity model and the dashed line represents results for Smagorinsky model. The notations for different models of LES are the same. As expected the results of the LES model without subgrid parametrization exhibits the largest deviation from DNS results. The application of SGS models significantly improves the accuracy of computations. In spite of good agreement of subgrid models with DNS data it is difficult to make the final conclusion which one of the models is the most effective for certain. The cross-helicity model give the best results, however the Kolmogorov model (demonstrating less accurate results in comparison with the Smagorinsky model and cross-helicity model) is the most advantageous in computational efforts. One more test for LES

provides the spectral distribution of the kinetic and the magnetic energies that shows redistribution of energy depending on wavenumber, i.e. at different scales. Fig. 3 and Fig. 4 present curves for kinetic and magnetic energy spectrums respectively at time $t = 0.5$.

It is clear that at large scales (small values of wavenumber) plots of curves practically coincide and dependence from subgrid parameterizations in fact

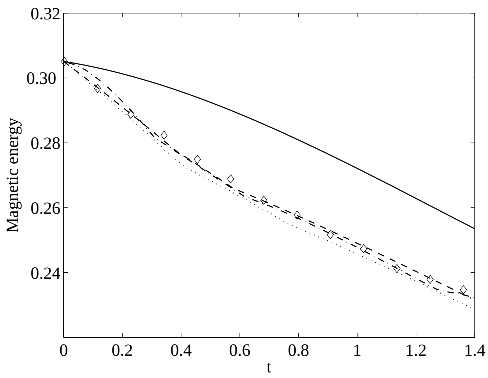

Fig. 2. Time evolution of the magnetic energy. Diamond line - DNS-results; solid line - LES results without subgrid closures; dot line - Kolmogorov model; dashed line - Smagorinsky model; dash-dot line - cross-helicity model.

disappears. Differences become apparent mainly at small scales (corresponding to large wave numbers). For the case of LES without subgrid modeling, the spectrum profile prove the necessity of application of the subgrid closures, because energy accumulates due to lack of dissipation supplied by the filtered out subgrid scale eddies.

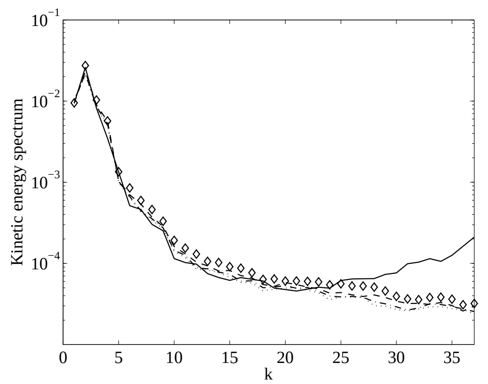

Fig. 3. Kinetic energy spectrum. Diamond line - DNS-results; solid line - LES results without subgrid closures; dot line - Kolmogorov model; dashed line - Smagorinsky model; dash-dot line - cross-helicity model.

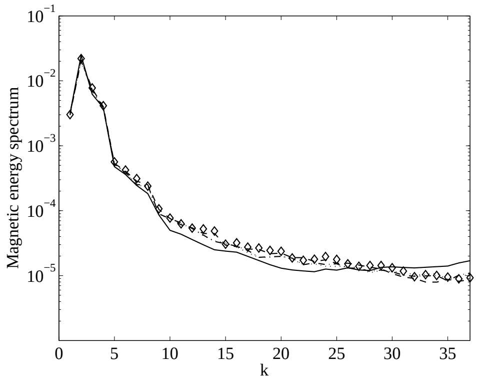

Fig. 4. Magnetic energy spectrum. Diamond line - DNS-results; solid line - LES results without subgrid closures; dot line - Kolmogorov model; dashed line - Smagorinsky model; dash-dot line - cross-helicity model.

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