

Stability study for quark systems using a semi-classical billiard model

Cristian C. Bordeianu¹, Călin Beșliu¹, Alexandru Jipa¹, Daniel Felea², Ioan V. Grossu¹, and Tiberiu Eșanu¹

¹ University of Bucharest, Faculty of Physics
Bucharest-Magurele, PO Box MG 11, 77125, ROMANIA
(e-mail: cristian.bordeianu@brahms.fizica.unibuc.ro)

² Institute of Space Sciences, Laboratory of Space Research
Bucharest-Magurele, PO Box MG 23, 77125, ROMANIA
(e-mail: felea@venus.nipne.ro)

Abstract. We consider a semi-classical version of the motion of the particles and of the collective coordinates. Several interacting quarks are moving in a 2D potential well and hitting the vibrating surface. We use a Cornell type potential consistent with the indications of lattice QCD calculations. The numerical simulation is based on the solutions of the Hamilton equations which was solved using an algorithm of Runge-Kutta type (order 4-5) having an optimized step size, taking into account that the absolute error for each variable is less than 10^{-6} . Total energy is conserved with high accuracy, i.e. approx. 10^{-6} in absolute value. We analyze the chaotic behavior of the nonlinear dynamics system using phase-space maps, autocorrelation functions, power spectra and Lyapunov exponents. There is a transition from a chaotic motion corresponding to hadrons plasma to a more ordered behaviour of the system which could be revealed from the dependence of largest Lyapunov exponent (LLE) versus temperature.

Keywords: Lyapunov exponent, Quark-gluon plasma.

1 Introduction

Over the last two decades an increasing number of papers have treated the study of the deterministic chaotically behaviour of Fermi nuclear systems. The relation between the order to chaos transition in the dynamics of independent classical particles in a container was first studied using computer simulations by Blocki et al [4]. They analyzed the behaviour of a gas of classical non - interacting particles enclosed on a multipole - deformed container which undergoes periodic shape oscillations and showed that higher multipolarities leads to chaotic motion. Another step in this direction was done by Bauer et al. in [2]. They have performed self-consistent calculations in semi classical approximation utilizing a multipole-multipole interaction of the Bohr-Mottelson type for quadrupole and octupole deformations. In both cases the dynamical evolution showed a regular undamped collective motion which coexist with a weakly chaotic single-particle dynamics. Then Burgio et al. [6] considered a number of nucleons without spin and charge and with

no internal structure which are moving in a 2D deep Woods-Saxon potential well and hitting the oscillating surface with a certain frequency.

In this paper we use a very similar model in order to study the critical behaviour of quarks undergoing phase transition to hadrons. This issue was previously studied in the framework of the Ising model [9]. Other studies [3] analyzed the largest Lyapunov exponents of SU(1) and SU(2) gauge field configurations on the lattice which are initialized by quantum Monte Carlo simulations. The results demonstrate that chaos is present when particles are confined, but it persists also into the Coulomb and quark-gluon plasma phase.

2 Toy Model

We chose the system under study as an ensemble with N quarks interacting one with each other only via a 2D Cornell potential well and hitting periodically the oscillating surface with a certain frequency. The Hamiltonian of this kind of interaction in polar coordinates is:

$$H(r_i, \theta_i, \alpha) = \sum_{i=1}^N \left(\frac{p_{r_i}^2}{2m} + \frac{p_{\theta_i}^2}{2mr_i^2} + V_{int}(r_i, R(\theta_i)) \right) + \frac{p_\alpha^2}{2M} + \frac{1}{2}M\Omega^2\alpha^2 \quad (1)$$

where $\{p_{r_i}, p_{\theta_i}, p_\alpha\}$ are the conjugate momenta of the i^{th} particle and of the collective coordinates $\{r_i, \theta_i, \alpha\}$, $m = 300MeV$, Ω is the oscillating frequency of the collective variable α and $M = mNR_0^2$ is the Inglis mass.

The colored and flavored quarks of u and d types are treated as semi classical particles interacting via a Cornell potential with color matrices C_{ij} , where V_{int} is given by the function:

$$V_{int} = \frac{1}{2} \sum_{i,j} C_{ij} V_C(|r_i - r_j|) \quad (2)$$

V_C being the Cornell potential given by the function:

$$V_C(r_i, R(\theta_i, \alpha)) = \sigma(r_i - R(\theta_i, \alpha)) - \frac{\alpha_0}{r_i - R(\theta_i, \alpha)} \quad (3)$$

where $\sigma = 900MeV/fm$ is the string tension and $\alpha_0 = 1.25$ is the QCD fine structure constant [1].

This potential is phenomenological motivated to mimic the soft gluon part of a quark-gluon plasma in accord with qMD model [12]. In Table 1 we present the elements of the color matrices for different elementary color combinations of the quarks. The quarks were initially random distributed in the billiard having radius R_0 and the corresponding momenta were generated according to Maxwell distribution. The vibrating surface of the potential well can be

C_{ij}	R	G	B	\bar{B}	\bar{G}	\bar{R}
R	-1	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	+1
G	$+\frac{1}{2}$	-1	$+\frac{1}{2}$	$-\frac{1}{2}$	+1	$-\frac{1}{2}$
B	$+\frac{1}{2}$	$+\frac{1}{2}$	-1	+1	$-\frac{1}{2}$	$-\frac{1}{2}$
\bar{B}	$-\frac{1}{2}$	$-\frac{1}{2}$	+1	-1	$+\frac{1}{2}$	$+\frac{1}{2}$
\bar{G}	$-\frac{1}{2}$	+1	$-\frac{1}{2}$	$+\frac{1}{2}$	-1	$+\frac{1}{2}$
\bar{R}	+1	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	-1

Table 1. Color matrix elements of different elementary color combinations of the quarks [12].

written as usually [7] depending on the collective variable and Legendre polynomials $P_L(\cos(\theta_i))$:

$$R(\theta_i, \alpha) = R_0[1 + \alpha \cdot P_L(\cos(\theta_i))] \quad (4)$$

where $R_0 = 6.0 \text{ fm}$ and L=0 (monopole), L=1 (dipole), L=2 (quadrupole), L=3 (octupole).

The numerical simulations are based on the solution of a system containing $4N + 2$ nonlinear Hamilton equations coupled by the collective variable α :

$$\dot{r}_i = \frac{p_{r_i}}{m} \quad (5)$$

$$\dot{p}_{r_i} = \frac{p_{\theta_i}^2}{mr_i^3} - \frac{\partial V_{int}}{\partial r_i} \quad (6)$$

$$\dot{\theta}_i = \frac{p_{\theta_i}}{mr_i^2} \quad (7)$$

$$\dot{p}_{\theta_i} = -\frac{\partial V_{int}}{\partial R} \cdot \frac{\partial R}{\partial \theta_i} \quad (8)$$

$$\dot{\alpha} = \frac{p_\alpha}{M} \quad (9)$$

$$\dot{p}_\alpha = -M\Omega^2\alpha - \frac{\partial V_{int}}{\partial R} \cdot \frac{\partial R}{\partial \alpha} \quad (10)$$

We performed the solving of the system of ordinary differential equations using an algorithm of Runge-Kutta type (order 4-5) having an optimized step size [10,11] and taking into account that the absolute error for each variable is less than 10^{-6} .

We developed our codes using Scilab 3.1.1. Scilab [5], developed by researchers from INRIA and ENPC, is a scientific software package for numerical computations providing a powerful open computing environment for engineering and scientific applications.

We choose Scilab, because provides various functions for ordinary differential equation solving, Fast Fourier Transform, autocorrelation and excellent 2D and 3D graphical capabilities.

3 Numerical study and preliminary conclusions

The chaos analyses we performed include phase portraits, power spectra, autocorrelation functions and Lyapunov exponents. We study systems having the number of quarks between 2 and 10.

One of the various ways for studying the dynamics of the system is to calculate the largest Lyapunov exponent (LLE) which is a measure of the sensitivity of the system to initial conditions.

The LLE was previously used to study the solid to liquid phase transitions in Lennard - Jones drops by Dorso and Bonasera in [8]. They showed the presence of a maximum in LLE which indicates the transition from a chaotic regime to a more regular one. In the chaotic regime the system is composed mainly of a liquid drop while the regular one corresponds to almost freely flowing particles and small clusters.

We calculate the multi-dimensional Lyapunov exponents using the formula:

$$\lambda(t) = \lim_{d_0 \rightarrow 0} \ln \frac{\sqrt{\sum_{i=1}^N [(r - r_0)^2 + (\theta - \theta_0)^2 + (p_r - p_{r_0})^2 + (p_\theta - p_{\theta_0})^2]_i}}{d_0} \quad (11)$$

where the sums are taken over all N quarks of the system and the quantities from the parentheses refer to two trajectories in the quark-quark phase-space which differ only by an infinitesimal quantity $d_0 = 10^{-6}$. We take integrations times about to $1500 fm/c$ sufficiently large to avoid errors on the calculation of the Lyapunov exponents.

In Fig. 1 we present the dependence of multi-dimensional Lyapunov exponent versus time. It can be seen the saturation behavior and the appearance of a plateau. The straight line represents a fit whose slope is the largest Lyapunov exponent (LLE). The preliminary results for two quarks systems are presented in Figure 2. All the calculations are done for $L = 2$ (dipole). It can be seen that LLE's have positive values, which is an evidence of a chaotically behaviour of the system.

Moreover we believe that the dependence of the Lyapunov exponents versus temperature can give us qualitative and quantitative information about a phase transition from the hadrons plasma to quark-gluon plasma. Thus the presence of the maximum in the LLE signals a transition from a chaotic motion corresponding to hadrons plasma to a more ordered behaviour of the

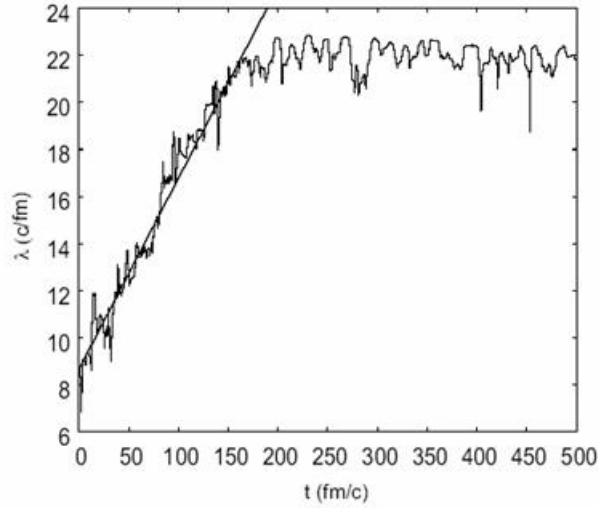


Fig. 1. The multi-dimensional Lyapunov exponent as a function of time for two quarks system. The straight line represents a fit whose slope equals the largest Lyapunov exponent (LLE).

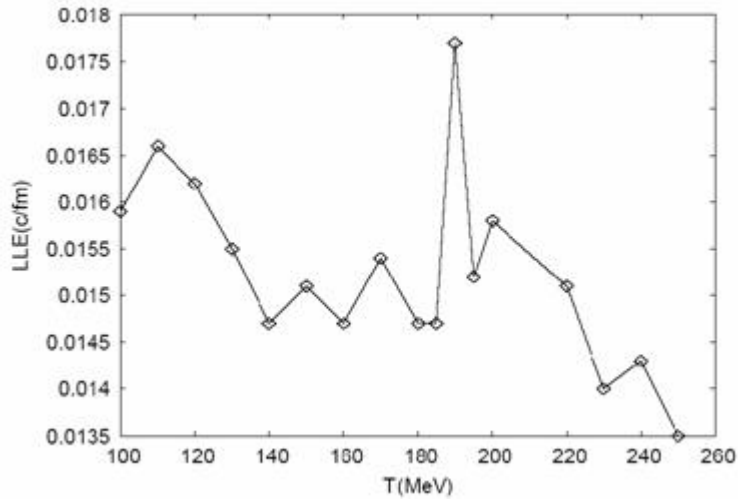


Fig. 2. The largest Lyapunov exponent (LLE) as a function of temperature for two quarks system. (The lines which connect points are only for eye orientation).

system, which could be associated with the above mentioned phase transition.

A similar correlation was made by using Monte Carlo numerical simulations for nucleonic systems by Zhang, Wu and Lee in [13]. Accordingly, the temperature corresponding to the maximal LLE is named "critical temperature" which signals a transition from a chaotic to a more ordered motion.

We believe that future works considering an increasing number of quarks will offer valuable information about the phase transition to quark-gluon plasma.

References

- 1.Y. Akimura, T. Maruyama, N. Yoshinaga, and S. Chiba. Molecular dynamics simulation for the baryon-quark phase transition at finite baryon density. *Eur. Phys. J. A*, 25:405–411, 2005.
- 2.W. Bauer, D. McGrew, V. Zelevinsky, and P. Schuck. Coexistence of regular undamped nuclear dynamics with intrinsic chaoticity. *Phys. Rev. Lett.*, 72:377, 1994.
- 3.T. S. Biro, N. Hormann, H. Markum, and R. Pullirsch. Chaos analyses in both phases of qed and qcd. *hep-ph/9909309*, 1999.
- 4.J. Blocki, J. Shi, and W.J. Swiatecki. Order, chaos and nuclear dynamics. *Nucl. Phys. A*, 554:387, 1993.
- 5.C. C. Bordeianu, C. Besliu, Al. Jipa, D. Felea, and I. V. Grossu. Scilab software package for the study of dynamical systems. *Computer Physics Communications*, DOI 10.1016/j.cpc.2008.01.002, 2008, 2008.
- 6.G. F. Burgio, F. M. Baldo, A. Rapisarda, and P. Schuck. Chaoticity in vibrating nuclear billiards. *Phys.Rev. C*, 52:2475, 1995.
- 7.A. deShalit and H. Feshbach. *Theoretical Nuclear Physics*, volume 1. Wiley, New York, 1974.
- 8.C. O. Dorso and A. Bonasera. Lyapunov exponent, generalized entropies and fractal dimensions of hot drops. *Eur. Phys. J. A*, 11:421–426, 2001.
- 9.R. C. Hwa. Critical and chaotic behavior of quarks. *hep-ph/9611372v2*, 1996.
- 10.R.H. Landau, M.J. Paez, and C.C. Bordeianu. *Computational Physics. Problem Solving with Computers*. Wiley VCH, 2 edition, 2007.
- 11.R.H. Landau, M.J. Paez, and C.C. Bordeianu. *A Survey of Computational Physics. Introductory Computational Science*. Princeton University Press, 2008.
- 12.S. Scherer. *Modeling Ultra-relativistic Heavy Ion Collisions with the quark Molecular Dynamics qMD*. PhD thesis, University Johann Wolfgang Goethe Frankfurt, 2005.
- 13.Y. Zhang, X. Wu, and Z. Li. Connection between the largest lyapunov exponent, density fluctuation, and multifragmentation in excited nuclear systems. *Phys. Rev. C*, 69:044609, 2004.