Measurement and Simulation of Random Matrix Statistics in Aluminum Mesoscopic Cavities

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Abstract: We investigated spectral fluctuations in mesoscopic aluminum cavities for acoustic waves with and without of reflection plane symmetry.
Results of measurements and simulations of cavity responses are compared to the predictions of random Hamiltonian model with time reversal symmetry.

Keywords: acoustic waves, mesoscopic cavity, spectral fluctuation, time-reversal symmetry

1. Introduction

Responses of aluminum cavities to short excitation pulses were recorded for different positions of the source and receiver. Sequences of resonance frequencies were obtained from spectral density of the recorded responses. These sequences were used to calculate nearest neighbor resonance spacing distributions (NNSD) and spectral rigidity (SR), parameter [1,2] characterizing fluctuation of number of resonances per frequency band. Exact model representations of the cavities were simulated with the program Wave3000 [3]. So the NNSD and SR were also calculated from simulated acoustic wave dynamics. Results are compared to predictions [1,2] of Random Matrix Theory (RMT) for random Hamiltonian with time reversal symmetry. These predictions [1,2] are valid for the eigenvalues of an ensemble of random symmetric matrices with real Gaussian distributed elements, or Gaussian Orthogonal Ensemble (GOE).

2. The Samples

The samples used in measurement and simulation are shown in figures 1 and 2. The shapes of the samples are cubic with the side length of the cube $d = 2$ cm. Cylindrical volume of aluminum is removed from each sample (figures 1, 2) as compared to the perfect cube. In addition Cavity Nr. 2 has a prism of aluminum removed from the edge.

Sound velocities for the aluminium used can be estimated as 6.4 km/s and 3.2 km/s for longitudinal and transverse waves respectively [4]. Therefore the corresponding wavelengths ranges of the frequency band 200..450 kHz used in the measurement are approximately $1.6d..0.7d$ in wavelength of the longitudinal wave and 0.8*d*..0.35*d* in wavelength of transversal wave.

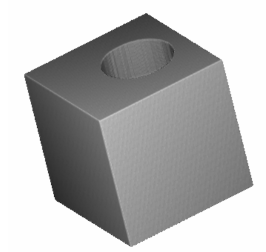

Figure 1. Cavity Nr. 1, with reflection symmetry.

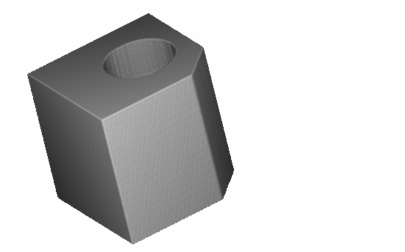

Figure 2. Cavity Nr. 2, without reflection symmetry.

3. Measurements and Simulation

The schematic of experimental setup is shown in figure 3. Sending the exciting pulse and receiving the cavity response is done by piezoelectric sensors, connected to the sample. Amplified response signal is recorded by the digitizing oscilloscope. The recorded 5 ms responses with a time step of 0.1 μs are used further for the calculation of the spectral density and determing sequences of cavity resonances detected at current position of source and receiver.

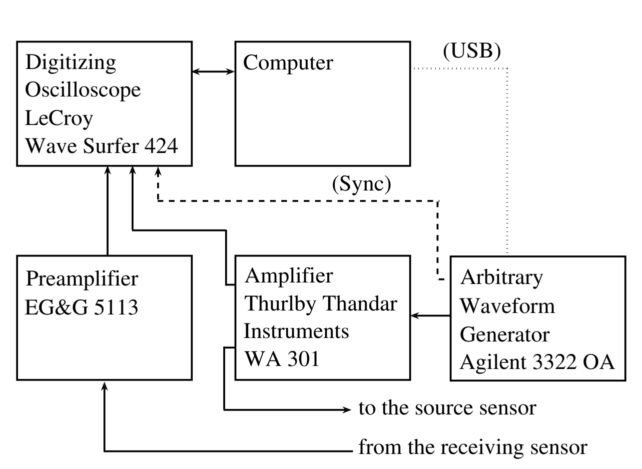

Figure 3. Experimental setup.

The same procedure is repeated for different positions of source and receiver. Spectral statistics (NNSD and SR) are calculated for each sequence of resonances and averaged over different sequences corresponding to different positions of source and receiver.

We also performed simulations of the cavity responses using the program Wave3000 [3] that has been developed by Kaufman et al. to calculate ultrasonic responses in complex solid materials using an adapted finite difference method. Figure 4 shows an example of the results for the measured and the simulated response. The details of the cavity geometries, the coupling to the environment, and the exact response of the transducers are too complex to expect a detailed match between experiment and simulation. In particular the reverberation time is much shorter in the experimental results. However, the statistcal properties of the spectra should be the same.

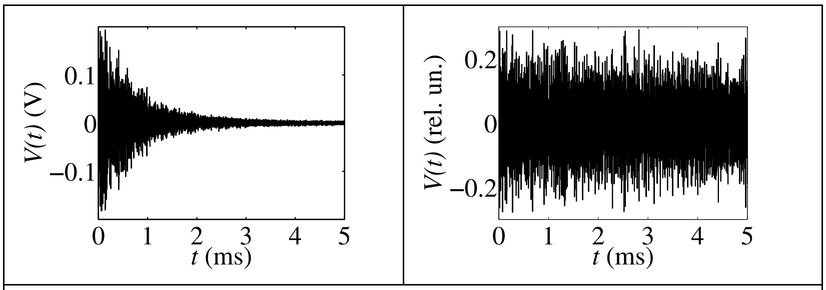

Figure 4. Measured (left figure) and simulated (right figure) cavity responses at certain positions of the source and receiver.

4. Discussion of results

4.1. Staircase function from the experiment

Staircase function *N(f)* gives the number of resonances in the studied frequency band below frequency *f*. It represents the sequence of cavity resonances detected at certain fixed position of source and receiver. Typical staircase functions obtained from the measurement are shown in figure 5.

4.2. NNSD and SR determined from the experiment

The NNSD from data measured in experiment for cavity Nr. 2 is shown figure 6. Dotted line shows exponential distribution (as for the case of spacing between “randomly chosen” resonant frequencies in the studied band, Poisson distribution). Dashed line shows the distribution corresponding to the GOE model. We include in our analysis the effect of missing resonance frequencies in the experimental detection. The dash dotted line shows the same GOE distribution when 25% of the resonances in the sequence are not detected. Points with error bars show the NNSD from the experimental data. The fluctuation shown by the error bars appears due to the different positions of source and receiver in the measurement.

The SR from data measured in experiment for cavity Nr. 2 is shown in figure 7. Dashed lines show the case of random fluctuation of the number of resonances per frequency band (straight line) and fluctuation corresponding to GOE model [1,2] (logarithmically saturating curve).

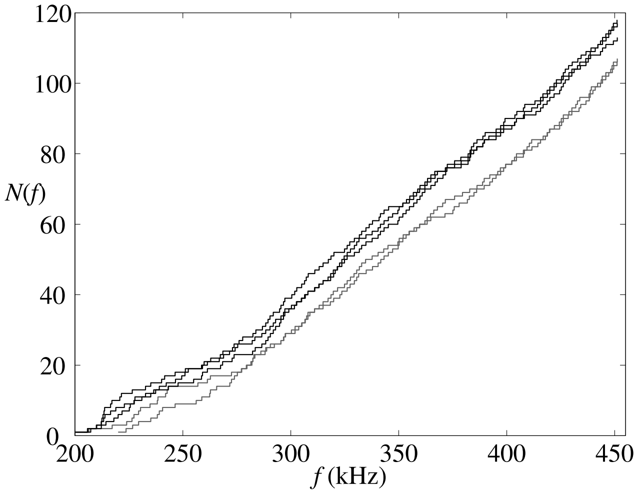

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Figure 5. Staircase functions obtained from the cavity responses measured in experiment.

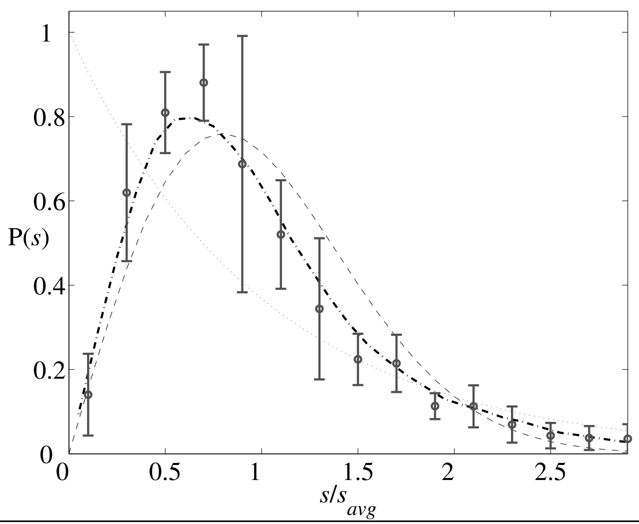

Figure 6. NNSD for cavity Nr. 2 from responses measured in experiment.

It is noticable that the points of the SR dependences for both cavities fall closer to the GOE prediction than to the line corresponding to a Poisson

NNSD. SR for cavity Nr. 1 (with reflection plane) is found slightly greater than for cavity Nr. 2. The difference may be caused by a larger ammount of lost resonances in cavity Nr. 1, which can be explained by the coexistance of two independent families of resonances (symmetric and antisymmetric) due to presence of reflection plane symmetry for cavity Nr. 1. Fluctuation shown by error bars for both cavities appears due to different position of source and receiver in the measurement.

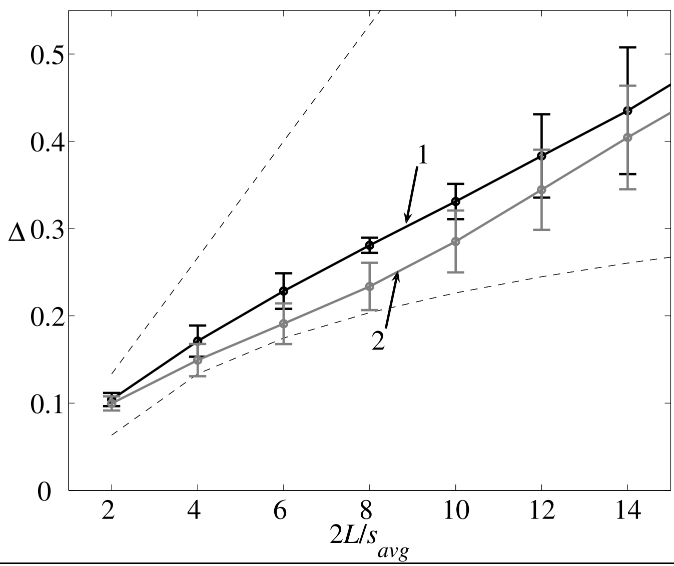

Figure 7. Spectral rigidity for both cavities Nr. 1 and 2 (shown by arrows in the picture). From responses measured in experiment.

4.3. NNSD and SR determined from simulation

NNSD calculated from simulated responses of cavity Nr. 2 is shown in figure 8. It does not show the behavior due to a fraction of resonances not detected, like in case of NNSD determined from experiment. This is visible because the peak of the distribution is not shifted to the smaller nearest neigbour spacings, like in figure 6 (data from experiment). However NNSD determined from simulation shows greater repelling of resonant frequencies than in the model distribution. We see that NNSD from simulation has higher peak value while both NNSD from simulation and GOE model distribution are normalized to the unit area under the curve.

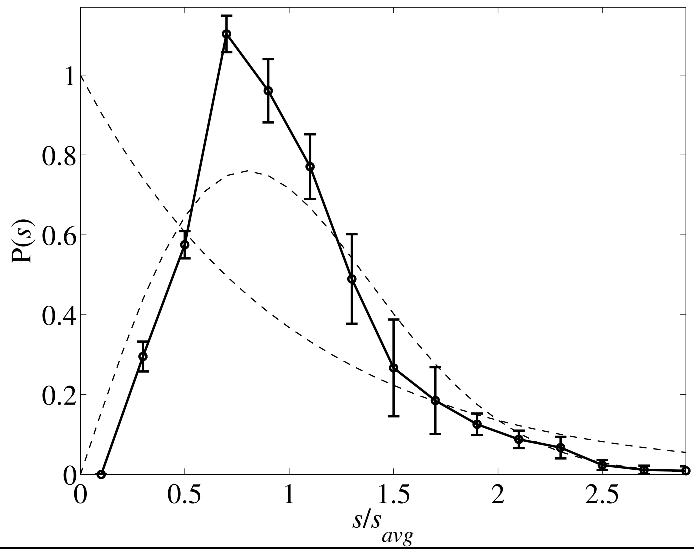

Figure 8. NNSD calculated for cavity Nr. 2 from responces obtained in Wave3000 simulation program for different positions of source and receiver.

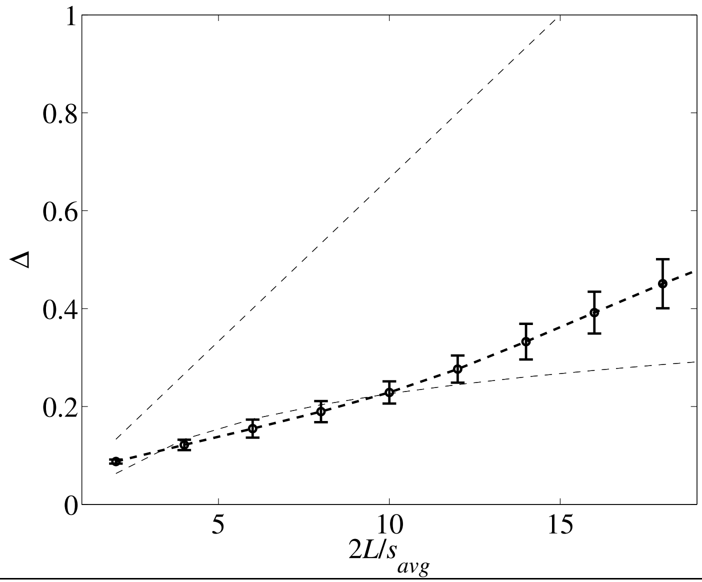

Figure 9. Spectral rigidity for cavity Nr. 2. From responses obtained in Wave3000 simulation program.

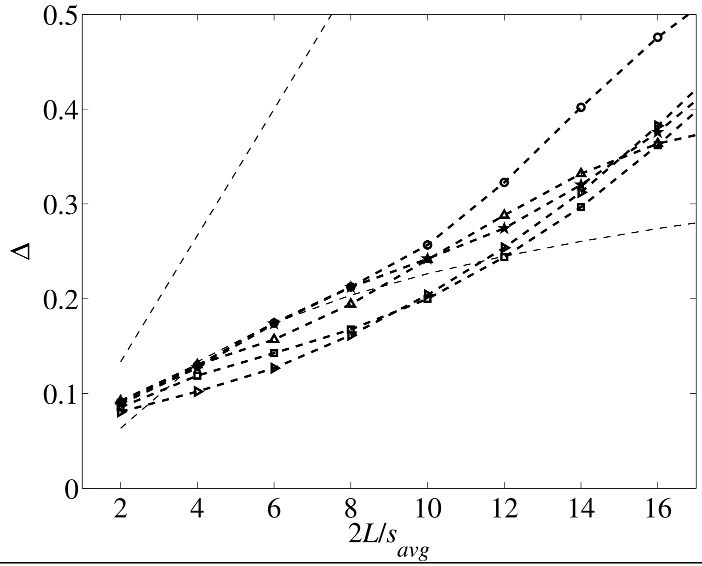

Figure 10. Spectral rigidity for cavity Nr. 2. From response obtained in Wave3000 simulation program. Different curves in this picture show results for different positions of source and receiver.

SR calculated from the simulation for cavity Nr. 2 is shown in figure 8.
It is also in agreement with GOE model. However unlike the dependence determined from experimental data SR determined from simulattion has lower values than predicted by the model. This is better explained by figure 9. SR calculated from simulation for each particular position of source and receiver is shown in figure 10. We see that these dependences either exactly coinside with GOE model for frequency bands < $10S_{avg}$ or take lower value.

5. Conclusions

The NNSD obtained from the measured responses agrees with the model of random Hamiltonian preserving the time-reversal symmetry. Agreement was achieved when accounting for a 25% of resonances lost in the sequence. Spectral rigidity obtained from the measured responses behaves according to the GOE model when averaged over frequency bands smaller than 10 average resonance frequency spacings S_{avg}.. The simulated acoustic wave dynamics shows greater ammount of resonances and noticeably lower spectral fluctuations. This is consistent with the longer reverberation times in the calculations. The NNSD is better peaked around average value and the SR averaged over frequency bands smaller than 10 S_{avg} has lower value than predicted by the GOE model.

6. Acknowledgements

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