A note on quantum chaology and gamma approximations to eigenvalue spacings for infinite random matrices

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Quantum counterparts of certain simple classical systems can exhibit chaotic behaviour through the statistics of their energy levels and the irregular spectra of chaotic systems are modelled by eigenvalues of infinite random matrices. We use known bounds on the distribution function for eigenvalue spacings for the Gaussian orthogonal ensemble (GOE) of infinite random real symmetric matrices and show that gamma distributions, which have an important uniqueness property, can yield an approximation to the GOE distribution. That has the advantage that then both chaotic and non chaotic cases fit in the information geometric framework of the manifold of gamma distributions, which has been the subject of recent work on neighbourhoods of randomness for general stochastic systems. Additionally, gamma distributions give approximations, to eigenvalue spacings for the Gaussian unitary ensemble (GUE) of infinite random hermitian matrices and for the Gaussian symplectic ensemble (GSE) of infinite random hermitian matrices with real quaternionic elements, except near the origin. Gamma distributions do not precisely model the various analytic systems discussed here, but some features may be useful in studies of qualitative generic properties in applications to data from real systems which manifestly seem to exhibit behaviour reminiscent of near-random processes.

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