Self-similarity/Memory-length Parameter Estimation for non-Gaussian Hermite Processes via Chaos Expansion

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A large number of phenomena in different scientific fields, such as economics, biology, medicine, traffic network, hydrolodgy exhibit long-range dependence, i.e. long memory. A class of stochastic processes, known as self-similar stochastic processes, can be used to effectively model such phenomena. The family of self-similar processes is parametrized by the self-similarity (or Hurst) index, which is in fact the parameter that describes the memory and consequently the behavior of those phenomena.
Hermite processes are a family of such processes that encompass the well-known fractional Brownian motion, and also include many non-Gaussian processes. Because of the wide applicability of Hermite processes in the modeling of many physical phenomena, a crucial question for researchers is to be able to estimate reliably the Hurst index of such processes using the available observations. However, a satisfying statistical method for this purpose does not exist. Our purpose in this work is to study the behavior of Hermite processes of order q with self-similarity index H from a theoretical point of view, and to develop a consistent estimator for the Hurst index, which will have an immediate impact on all aforementioned applied areas.
Our methodology is to use the variations of the filtered self-similar processes, for filter of order 1, as well as Malliavin calculus techniques in order to construct an estimator for the self-similarity index. We prove that these variations converge in $L^{2}(Omega);$ to a Rosenblatt random variable, which is an element of the second Wiener chaos. Moreover, in the case of the Rosenblatt process (Hermite process of order 2) we develop consistent estimators and study the behavior of the filtered process for different filter lengths, extending previous results for the fractional Brownian motion.
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References:
1.D. Nualart (2006). Malliavin Calculus and related topics, 2nd ed. Springer V.
2. D. Nualart and S. Ortiz-Latorre (2007). Central limit theorems for multiple stochastic integrals and Malliavin calculus. To appear in: Stochastic Processes and their Applications.
3. C.A. Tudor, F.G. Viens (2007): Variations and estimators for the selfsimilarity order through Malliavin calculus. Preprint.